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# IMPACT OF ELECTROMAGNETIC INTERFERENCES ON TRACTION NETWORK: MATHEMATICAL MODEL OF TRACTION NETWORK

## Introduction

The railway development has been significant since the beginning of the 20th century. The most significant components of the railway system are the trains. In the beginning, the power of the train was generated using steam and diesel. But, in 1897, Siemens presented the first electrically powered locomotive at the Berlin Commerce Fair. The maximum speed of the Siemens train was 13 km/h. Since then, the electrically powered trains have been improved. There are two types of railway electrical supply networks in Europe: direct current (DC), single-phase alternating current (AC). In the AC system, power is fed to the train by an overhead line (OHL). An OHL system is shown in Fig. 1. A third rail placed aside the railway track is usually used to feed the train in the DC system. An example of this type of system is shown in Fig. 2 [1, 2].



Fig. 1. An OHL system

There are three main categories under the AC power supply system, including the rail-return (RR) system, booster-transformer (BT) system, and auto-transformer (AT) system.

These three systems mainly differ in how the traction current is fed into the train and the return current leaving the train flows back to the power supply. The results of [2] have shown that the auto-transformer and booster-transformer systems can reduce the interferences in the signaling cable, compared to the rail-return system. But, the disadvantage of the booster-transformer system is the voltage drop along the overhead line [2].



Fig. 2. A third rail system

## Objective

This paper introduces a mathematical model of the railway network which can be helpful in analysis of the impact of different faults on the railway network.

## Overview of Electrical Railway System in Ukraine

In Ukraine, 47% of railway lines are electrified and 53% have diesel traction. Ukraine has 25 kV-50 Hz AC traction system with a

length of 5500 km like Russia, China, France, the UK, Hungary, Germany, Denmark, Italy, and Portugal. Also, Ukraine has 3kV DC traction system like Russia, Belgium, Spain, Italy, and Poland with a length of 5000 km. There are 25 Hz and 75 Hz track circuits used in AC traction systems and 50 Hz track circuits used in DC traction systems in Ukraine [3, 4]. The automatic locomotive signaling (ALS) system and automatic block systems are used to transfer the codes between the locomotive and receiving devices of the track circuit. In fact, they regulate the traffic of trains on the railway sections [5].

$$Z_{ii,ext} = j\omega \frac{\mu_0}{2\pi} \ln \frac{2h_i}{r_i} + 2(\Delta R_{ii} + j\Delta X_{ii}) \quad (1)$$

$$Z_{ij,ext} = j\omega \frac{\mu_0}{2\pi} \ln \frac{D_{ij}}{d_{ij}} + 2(\Delta R_{ij} + j\Delta X_{ij}) \quad (2)$$

### Electromagnetic Interferences in Railway System

There is a huge number of faults in automatic devices of the railway. These faults occur because of the stray currents, presence of harmonics which originated from locomotive's motors, communication devices, IGBTs, thyristors, etc., and impulse interferences on the return traction current. Statistics in Ukraine show that 9.2% of failures are due to the interferences of traction currents [4, 5].

Fig. 3 shows the block diagram of the AC traction power supply system with an AC motor. In this diagram, the AC is converted to DC by the rectifier. After passing the filter, the DC is converted to the 3-phase power by the inverter. Several stages between the power supply and the motor and unwanted electromagnetic disturbance (interferences) can be generated at any of the aforementioned steps. These interferences can be distributed in the cables and radiated through different loop circuits. The interferences are generated because the rectifier and inverter can extend to the megahertz scale [1].

It is evident that the complexity of the railway system increases with the increase of

electronics device usage. On the other hand, it is mentioned that the environment of the railway tracks is exposed the electromagnetic interferences from the traction power supply, power transmission lines, and other sources.

The presence of these electromagnetic interferences and more sensitive electronics devices make the railway system more sensitive and vulnerable [6].

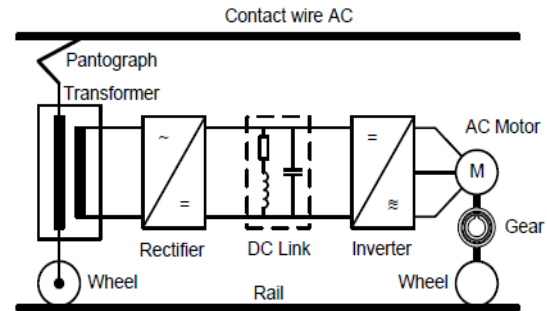


Fig. 3. Block diagram of the AC traction power supply system

As mentioned before, ALS systems are vital for safety and efficiency. These interferences can cause a false signal and cause an accident in railway systems. So, electromagnetic compatibility (EMC) becomes a very important issue in the railway system.

### Impact of EMI on Railway System

To investigate the electromagnetic interferences' impact on automatic devices of the railway system, firstly, all of the electromagnetic interference sources in the railway system should be known and investigated. Some works investigate the harmonics and pulses in traction currents in rail lines. For example, in [3, 7], the authors performed an experimental work and obtained the rail line current spectrum. The work in [7] was performed in DC and AC traction power supply systems. Secondly, all parts of the railway system should be modeled accurately. Some works investigate the model for traction net. Thirdly, the performance of automatic devices like track circuits should be investigated in presence of the electromagnetic interferences. In this re-

gard, the accurate model of the track circuits can be very helpful.

#### Mathematical Model of Traction Network

There are a number of mathematical models of the traction power supply system, in which the local load (electrical locomotive, consuming electrical energy from the contact network) is replaced by a uniformly distributed load, which is typical for rail sections with high intensity traffic working. In [8], the authors proposed a mathematical model for traction network to investigate the propagation of the harmonics in rail lines. This model consists of different impedances related to a different part of the network.

In electric traction, rail lines are used not only as a channel for the flow of the signal and the code currents, but also to allow a return of traction current. Code current, which flows in the rail circuit, is influenced by various sources of impacts. The proposed mathematical model is constructed for the case when the current consumption by electric locomotive, acting in a rail network, divides into the currents  $I_1$  and  $I_2$ , flowing in traction lines. In this case, part of the reverse traction current returns to the substation not by the rail lines, and through the ground. The magnitude of leakage current is determined by the conductivity of contours rail – rail, rail – ground, contact network – rails, and the contact network – the ground.

For the analysis of dynamic processes in electric circuits it is proposed to consider the rail circuit as a six-poles, and power train - as a 2n-poles. Such a representation of the circuit, which is shown in Fig. 4, takes into account the influence of various noise sources and effects of external factors that lead to a change in the parameters of the rail line and impact on work of related devices of railway automatics and remote control and communication lines.

### Results

For above mathematical model, system of differential equations (3) can be written, com-

posed by the methods of contour currents for internal and external contours of eight-poles, which have a transverse element (earth), and the base unit (b), and the method of nodal potentials.

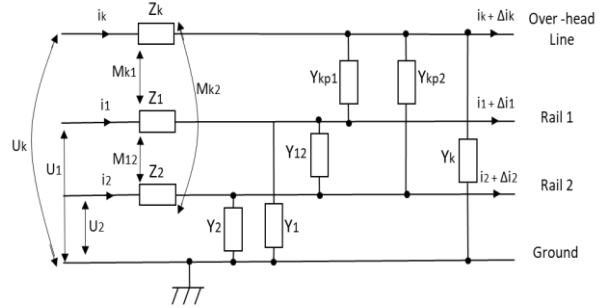


Fig. 4. Mathematical model of the traction network of a single-track section

$$\left\{ \begin{aligned}
 -\frac{d\dot{U}_k}{dx} &= \dot{I}_k \cdot \underline{Z}_k - \dot{I}_1 \cdot \underline{Z}_{M1k} - \dot{I}_2 \cdot \underline{Z}_{M2k}, \\
 -\frac{d\dot{U}_1}{dx} &= \dot{I}_1 \cdot \underline{Z}_1 - \dot{I}_k \cdot \underline{Z}_{M1k} - \dot{I}_2 \cdot \underline{Z}_{M12}, \\
 -\frac{d\dot{U}_2}{dx} &= \dot{I}_2 \cdot \underline{Z}_2 - \dot{I}_1 \cdot \underline{Z}_{M12} - \dot{I}_k \cdot \underline{Z}_{M2k}, \\
 -\frac{d\dot{U}_{12}}{dx} &= -\frac{d\dot{U}_1}{dx} + \frac{d\dot{U}_2}{dx}, \\
 -\frac{d\dot{U}_{k1}}{dx} &= -\frac{d\dot{U}_k}{dx} + \frac{d\dot{U}_1}{dx}, \\
 -\frac{d\dot{U}_{k2}}{dx} &= -\frac{d\dot{U}_k}{dx} + \frac{d\dot{U}_2}{dx}, \\
 -\frac{d\dot{I}_1}{dx} &= \underline{Y}_1 \cdot \dot{U}_1 + \underline{Y}_{12} \cdot (\dot{U}_1 - \dot{U}_2) - \underline{Y}_{kp1} \cdot (\dot{U}_1 - \dot{U}_k), \\
 -\frac{d\dot{I}_2}{dx} &= \underline{Y}_2 \cdot \dot{U}_2 + \underline{Y}_{12} \cdot (\dot{U}_2 - \dot{U}_1) - \underline{Y}_{kp2} \cdot (\dot{U}_2 - \dot{U}_k), \\
 -\frac{d\dot{I}_k}{dx} &= \underline{Y}_k \cdot \dot{U}_k + \underline{Y}_{kp1} \cdot (\dot{U}_k - \dot{U}_1) - \underline{Y}_{kp2} \cdot (\dot{U}_k - \dot{U}_2),
 \end{aligned} \right. \quad (3)$$

where  $\dot{U}_1, \dot{U}_2, \dot{U}_k, \dot{U}_{12}, \dot{U}_{k1}, \dot{U}_{k2}$  – are complex values of voltage drops across 1 km length contours: rail1 – ground, rail2 – ground, contact network – ground, rail 1 – rail 2, contact network – rail 1, contact network – rail 2,

respectively, V;  $\dot{I}_1, \dot{I}_2, \dot{I}_k$  – are complex values of current per 1 km in following contours: rail1 – ground, rail 2 – ground, contact network – ground respectively, A;  $\underline{Z}_1, \underline{Z}_2, \underline{Z}_k$  – are a complex resistivity in those contours respectively, Ohm/km;  $\underline{Z}_{M1k}, \underline{Z}_{M2k}, \underline{Z}_{M12}$  – are a complex resistivity of mutual induction in the contours: contact network – rail 1, contact network – rail 2, rail 1 – rail 2, respectively, Ohm/km;  $\underline{Y}_1, \underline{Y}_2, \underline{Y}_k, \underline{Y}_{12}, \underline{Y}_{kp1}, \underline{Y}_{kp2}$  – are a complex of conductivities in the contours: rail1 – ground, rail 2 – ground, contact network – ground, rail 1 – rail 2, contact network – rail 1, contact network – rail 2, respectively, S/km.

Conductivity between the overhead line and rails can be assumed to be zero because their value is very small [1-2, 10, 11]:

$$Y_{kp1} = Y_{kp2} = Y_k = 0. \quad (4)$$

Also, we can write these equations [1-2]:

$$Y_1 + Y_{12} = Y_{11} \quad \& \quad Y_2 + Y_{12} = Y_{22}. \quad (5)$$

Due to the symmetry of the rail lines, we can write [9-10]:

$$Z_{M1k} = Z_{M2k} = Z_M \quad (6)$$

Taking into account these equations we have:

$$-\frac{d^2 \dot{U}_1}{dx^2} = \dot{U}_1(-Z_1 Y_{11} - Z_{M12} Y_{12}) + U_2 \quad (7)$$

$$-\frac{d^2 \dot{U}_2}{dx^2} = \dot{U}_2(-Z_2 Y_{22} - Z_{M12} Y_{12}) + \dot{U}_1(Z_2 Y_{12} + Z_{M12} Y_{11}) \quad (8)$$

$$-\frac{d^2 \dot{U}_k}{dx^2} = \dot{U}_1(Z_M Y_1) + \dot{U}_2(Z_M Y_2) \quad (9)$$

Considering (5) and (6) as differential equation systems, we can write:

$$\frac{d^2}{dx^2} \begin{pmatrix} \dot{U}_1 \\ \dot{U}_2 \end{pmatrix} =$$

$$\begin{pmatrix} Z_1 Y_{11} + Z_{M12} Y_{12} & -Z_1 Y_{12} - Z_{M12} Y_{22} \\ -Z_2 Y_{12} - Z_{M12} Y_{11} & Z_2 Y_{22} + Z_{M12} Y_{12} \end{pmatrix} \begin{pmatrix} \dot{U}_1 \\ \dot{U}_2 \end{pmatrix}$$

The equation (8) is similar to the mass-spring system equation. This system can be written as:

$$\ddot{U} = -KU,$$

$$K = - \begin{pmatrix} Z_1 Y_{11} + Z_{M12} Y_{12} & -Z_1 Y_{12} - Z_{M12} Y_{22} \\ -Z_2 Y_{12} - Z_{M12} Y_{11} & Z_2 Y_{22} + Z_{M12} Y_{12} \end{pmatrix} \quad (10)$$

This system can be solved by guessing a form for the solution. We could guess:

$$U = ae^{wx} \quad (11)$$

Inserting (10) into the (9) gives:

$$(K - w^2 I)a = 0 \quad (12)$$

This is a homogeneous system. It is a generalized eigenvalue problem for eigenvalues  $w^2$  and eigenvectors  $a$ . We solve this in a similar way to the standard matrix eigenvalue problems. The eigenvalue equation is found as:

$$\det(K - w^2 I) = 0 \quad (13)$$

Once the eigenvalues are found, then one determines the eigenvectors and constructs the solution.

Solutions for this kind of differential equation can be classified into 3 cases [12].

*Case I: Two real, distinct roots*

Solve the eigenvalue problem  $Av = \lambda v$  for each eigenvalue, obtaining two eigenvectors  $\vec{v}_1, \vec{v}_2$ . Then write the general solution as a linear combination  $(x) = c_1 e^{\lambda_1 x} \cdot \vec{v}_1 + c_2 e^{\lambda_2 x} \cdot \vec{v}_2$ .

*Case II: One Repeated Root*

Solve the eigenvalue problem  $Av = \lambda v$  for one eigenvalue  $w^2$ , obtaining the first eigenvector  $\vec{v}_1$ . One then needs a second linearly independent solution. This is obtained

by solving the nonhomogeneous problem  $A \vec{v}_2 - \lambda \vec{v}_2 = \vec{v}_1$  for  $\vec{v}_2$ .

The general solution is then given by  $U(x) = c_1 e^{\lambda_1 x} \cdot \vec{v}_1 + c_2 e^{\lambda_2 x} \cdot (\vec{v}_2 + x \vec{v}_1)$ .

*Case III: Two complex conjugate roots*

Solve the eigenvalue problem  $Av = \lambda v$  for one eigenvalue  $\lambda = \alpha + i\beta$ , obtaining one eigenvector  $v$ . Note that this eigenvector may have complex entries. Thus, one can write the vector  $y(x) = e^{\lambda x} v = e^{\alpha x} (\cos \beta x + i \sin \beta x) v$ . Now, construct two linearly independent solutions to the problem using the real and imaginary parts of  $y(x)$ :  $y_1(x) = \text{Re}(y(x))$  and  $y_2(x) = \text{Im}(y(x))$ . Then the general solution can be written as  $U(x) = c_1 y_1(x) + c_2 y_2(x)$ .

Then,  $U(x) = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$  can be determined using one of the above cases which are related to the problem. So, having  $\dot{U}_1$  and  $\dot{U}_2$ , we can calculate the  $\dot{U}_k$  using equation (7). Now, having  $\dot{U}_1, \dot{U}_2$  and  $\dot{U}_k$ , we can calculate  $\dot{I}_1, \dot{I}_2$  and  $\dot{I}_k$  using (1).

### Conclusion

As mentioned before, one of the main steps to investigate the electromagnetic interferences' impact on automatic devices of the railway system is the accurate model of traction network. This paper introduced a mathematical model for traction network. The proposed model can be used when the electric locomotive current divides into two different current which flow in traction lines. Moreover, a system of differential equation regarding the proposed model was written and a solution to this system was introduced. All rail currents and the return current can be calculated from the above solution. Therefore, the impact of electromagnetic interferences on these current can be investigated.

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**Ключові слова:** система тягового електропостачання, рейкові кола, експериментальні дослідження, гармонійні завади.

**Ключевые слова:** система тягового электроснабжения, рельсовые цепи, экспериментальные исследования, гармонические помехи.

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