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MODELING OF THE TRACTION CURRENT HARMONICS DISTRIBUTION IN RAILS

Introduction

Electrified railways are one of the most powerful wide frequency range sources of disturbances that interference on signaling and radio communication systems. This is especially true for new types of vehicles equipped with electronic static converters with pulse width modulation (PWM), which high-frequency interferences in rails can have frequencies up to several tens of kHz. The electrical disturbances from vehicles (trains, locomotives) propagate through traction network [1].

There are five feeding systems that can be used with an AC electric railway: the direct feeding system, the direct feeding system with return wire, the booster transformer feeding system, the autotransformer feeding system and the coaxial cable feeding system [2, 3].

In general, the traction network are composed of the overhead supply conductors and traction rails electrically connected to earth or (and) with additional earth potential conductors. The return current from rolling stock axles flows in rails back to the supply substation. Therefore rails act as a path for electrical disturbance propagation.

To ensure the electromagnetic compatibility (EMC) of new types of rolling stock with signaling and radio communication systems, they are subjected to an acceptance procedure that includes on EMC tests in accordance with European and national regulatory standards and norms [1, 4].

But under some unfavorable operation conditions for the trains that were successfully tested and are in operation with old vehicles on the same lines (sometimes with old feeding

system), the disturbing current generated by vehicles may reach values greater than the allowed values [4, 5]. As such unfavorable operational conditions may be considered increasing number of trains, low distance between track circuit receiver and vehicles or supply substation, low rail-to-earth conductivity and conductivity of earth. To proof the electromagnetic compatibility (EMC) between rolling stock and signalling systems it is need accurate modeling of the test cases with taking into account particular operation conditions [1].

The models for the distribution of the traction return current in the direct feeding AC and DC electric railways have been described in many works (see for example [2]). The multi-conductor transmission line (MTL) approach has been successfully used in [6, 7] to investigate the electromagnetic processes in Auto Transformer (AT) electric traction systems for high speed railways.

The distribution of the traction return current harmonics in rails and their influence on track circuits have been modeled in [8,9]. To describe various traction feeding networks in a uniform way the chain circuit model was used in [3]. Harmonic power flow in traction network was analyzed in [10].

The aim of the work is to establish mathematical and computer model for distribution of traction current harmonics in AC direct feeding traction network with multiple vehicles in feeder zone. This model is an evolution and simplification of models represented earlier [8].

The work has been performed in order to proof the electromagnetic compatibility of new trains equipped with electronic static convert-

ers with existing traction lines and has been used during tests of new types of train [5].

Arrangement of the Traction network

The single-phase 1×25 kV and AT feeding 2×25 kV traction systems are commonly used in modern railway AC electrification system, including high-speed railways. The direct feeding single-phase 1×25 kV traction system with two tracks that have been intended for new type of trains operation were considered in the work.

The traction supply current flows from the electric supply substation (ESS) through the overhead supply conductors (messenger and contact wires). The traction return current flows back from the rolling stock axles to the supply station through the rails and the earth [2]. The return current divides among the rails and it differential current that is caused by electrical asymmetry of rails produces the disturbing current through the track circuit's receiver. If the frequencies of traction current harmonics are in the frequency range of the receiver's input filters the harmonics can cause failures in track circuits (TCs) operation. To obviate TCs failures the severe requirements are imposed onto the spectrum components of vehicle return current and also onto arrangement of the path for return current flow (cross bonds, earthing of masts, electrical asymmetry of rails, rail-to-earth conductance, etc.).

The track circuits considered in work have various designs, parameters of transmitter and receiver, operational frequencies (25, 420, 480, 580, 720, 780 Hz). The length of audio-frequency track circuits is $10^2..10^3$ m and TCs with 25 Hz signal current – up to 2.5 km.

Parameters of traction network

Spacious non-uniformities having extensive multitrack arrangement are the railway stations, overtaking stations, block-posts, and etc. Point discontinuities in railway arrangement may be related to electric locomotives, power transformers, track transformers, trans-

mitters and receivers of TCs, all sort of power devices along a railway line, tracks cross-bonds, insulating joints, earthings of masts connected to rails, short-circuits connections along the catenary, and etc. In the model such point discontinuities are represented as elements with lumped parameters connected to lines.

For modeling longitudinally non-uniform traction network it is usually divided on segments, which can be considered as homogeneous. So, the network is represented as a set of connected multipole segments, which numbers of input and output poles are dependent on the numbers of lines in segments. Direct feeding system has at least five lines per one track [2].

There have been made some simplifications for considered model. The lines with equal or close to each other potentials are represented as single line with equivalent electrical parameters and placement. In such way the overhead system that consists of contact and messenger wires is presented as single line with equivalent impedance per unit length (p.u.l.) [2]

$$\underline{Z}_{OV} = \frac{\underline{Z}_K \underline{Z}_M - \underline{Z}_{KM}^2}{\underline{Z}_K + \underline{Z}_M - 2\underline{Z}_{KM}}, \quad (1)$$

were \underline{Z}_K , \underline{Z}_M are impedances (p.u.l) of the contact wire and the messenger wire, respectively, \underline{Z}_{KM} is mutual impedance (p.u.l.) between the contact and the messenger wires.

The equivalent circuit of infinitesimal segment of length dx for two-track traction system is shown in fig. 1.

The lines with indices 1 and 2 corresponds to the rails of first track, 3, 4 – to the rails of second-track, 5, 6 – to the equivalent overhead wires of the first and second tracks, respectively, and the line 0-corresponds to the ground conductivity. The voltage $\dot{U}_i(x)$ corresponds to the voltage of i -th line relative to the ground, $\dot{I}_i(x)$ is the current in i -th line; $\underline{Z}_{ii} = R_i + jX_i = R_i + j\omega L_i$ is the intrinsic impedance (p.u.l.) of i -th line (without taking

into account inductive influence of other lines), R_i , X_i – active and reactive resistance (p.u.l.) of i -th line, L_i is inductance (p.u.l.) of i -th line, $\underline{Z}_{ij} = jX_{ij} = j\omega M_{ij}$ is the mutual impedance (p.u.l.) between i -th and j -th lines, M_{ij} is the coefficient of mutual inductance (p.u.l.) between i -th and j -th lines, $j = \sqrt{-1}$, $\omega = 2\pi f$ – cyclic frequency; Y_{ij} is the admittance between i -th and j -th` lines.

The mutual inductance between lines of first and second tracks isn't shown in fig. 1.

Intrinsic impedance of the line consists of three components: internal line impedance \underline{Z}_{Cii} , external line impedance \underline{Z}_{Eii} and impedance introduced by ground [12-15]

$$\underline{Z}_{ii} = \underline{Z}_{Cii} + \underline{Z}_{Eii} + \underline{Z}_{Gii}. \quad (2)$$

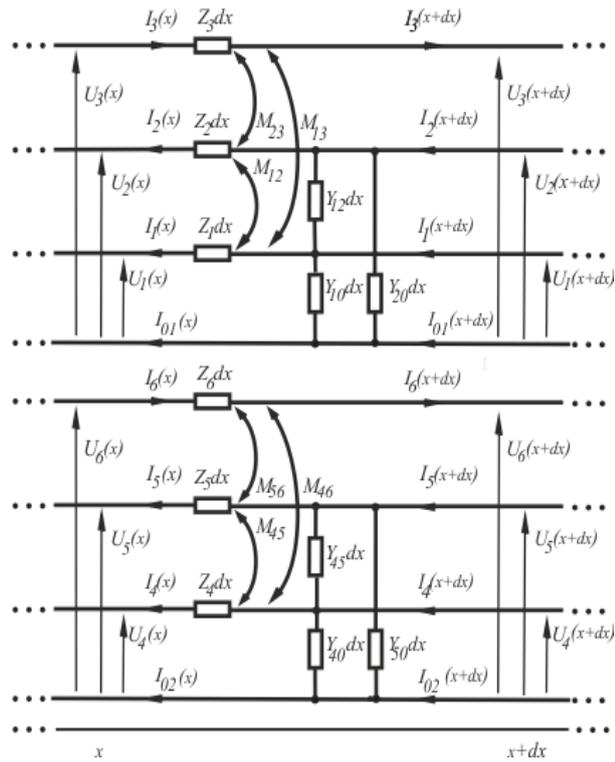


Fig. 1. The equivalent circuit of infinitesimal segment of length dx for two-track traction system

The internal impedance of the rail \underline{Z}_{Cii} ($i = 1, 2, 4, 5$) is determined taking into account the skin effect in the rail steel [16]

$$\underline{Z}_{Cii}(\omega) = R_{Cii}'(T) + R_{Cii}''(\omega) + jX_{Cii}(\omega), \quad (3)$$

where $R_{Cii}'(T)$ and X_{Cii} –the active and reactive resistance (p.u.l.) of the rail at a given temperature [16]

$$R_{Cii}''(\omega) = \frac{1}{u} \sqrt{\mu_0 \mu_s \rho_s \omega}, \quad (4)$$

$$X_{Cii} = \frac{0.6}{u} \sqrt{\mu_0 \mu_s \rho_s \omega}, \quad (5)$$

in which μ_s and ρ_s are, respectively, the magnetic permeability and conductivity of rail steel, u is the equivalent perimeter of the rail section.

The expressions for intrinsic impedance of i -th line \underline{Z}_{Gii} and the mutual impedance between i -th and j -th lines \underline{Z}_{Gij} in the presence of the earth as a return wire have been obtained by Carson [12]:

$$\underline{Z}_{Gii} = \frac{j\omega\mu}{2\pi} \ln\left(\frac{2h_i}{r_i}\right) + \underline{Z}'_{Gii}, \quad (6)$$

$$\underline{Z}_{Gij} = \frac{j\omega\mu}{2\pi} \ln\left(\frac{\sqrt{(h_i + h_j)^2 + d_{ij}^2}}{\sqrt{(h_i - h_j)^2 + d_{ij}^2}}\right) + \underline{Z}'_{Gij}, \quad (7)$$

$$\underline{Z}'_{Gii} = \frac{j\omega\mu}{\pi} \int_0^\infty \frac{\exp(-2h_i\xi)}{\xi + \sqrt{(\xi^2 + j\omega\mu\sigma)}} d\xi, \quad (8)$$

$$\underline{Z}'_{Gij} = \frac{j\omega\mu}{\pi} \int_0^\infty \frac{\exp[-(h_i + h_j)\xi]}{\xi + \sqrt{(\xi^2 + j\omega\mu\sigma)}} \cos(\xi d_{ij}) d\xi,$$

in which h_i, h_j are the heights of the wires above the ground, r_i is the radius of the wire, d_{ij} is the horizontal distance between the projections i -th and j -th lines, μ_0 is the magnetic constant.

The Carson's expressions for \underline{Z}'_{Gii} and \underline{Z}'_{Gij} include infinite integrals with complex arguments. It has been suggested to use for their approximation the infinite series with the restriction of the numbers of terms in the se-

ries. The Carson's expressions are widely used to determine intrinsic impedance of the line above the ground [3, 6-11].

In this work the approximations proposed in [14] for the intrinsic and mutual impedance of conductors above the earth's surface as a return wire have been used

$$\underline{Z}_{Gii} = \frac{j\omega\mu_0}{2\pi} \ln \left[\frac{2(h_i + p)}{r_i} \right], \quad (9)$$

$$\underline{Z}_{Gij} = \frac{j\omega\mu_0}{2\pi} \ln \left(\frac{\sqrt{(h_i + h_j + 2p)^2 + d_{ij}^2}}{\sqrt{(h_i - h_j)^2 + d_{ij}^2}} \right), \quad (10)$$

where $p = (j\omega\mu_0\sigma)^{-1/2}$ is the complex depth of the earth layer which the return current flows, σ is the earth conductivity.

Mathematical model

The mathematical model of the traction network has been performed using a Multi-conductor Transmission Line (MTL) approach with description of currents and voltage distributions in lines with equations in matrix form. The series impedances (p.u.l.) of the lines $\underline{Z}(f)$ and the shunt admittances between the lines (p.u.l.) $\underline{Y}(f)$ are represented as rectangular matrices of the size $n \times n$. The matrixes $\underline{Z}(f)$ and $\underline{Y}(f)$ are always symmetric. The elements of matrixes are calculated on the assumption that the dominant mode of wave propagation is transverse electromagnetic (TEM) in the dielectric and transverse magnetic (TM) in conductors, and that contributions from other modes are negligible.

The diagonal elements \underline{Z}_{ii} of the impedance matrix correspond to the intrinsic impedance (p.u.l.) of the i -th line without taking in account of the mutual inductive impedance. Off-diagonal elements \underline{Z}_{ij} correspond to the mutual impedance between i -th and j -th lines ($\underline{Z}_{ij} = \underline{Z}_{ji}$).

The diagonal elements of the admittance matrix are defined as

$$Y_{11} = -(Y_{10} + Y_{12}), \quad (11)$$

$$Y_{22} = -(Y_{20} + Y_{21}), \quad (12)$$

$$Y_{44} = -(Y_{40} + Y_{45}), \quad (13)$$

$$Y_{55} = -(Y_{50} + Y_{54}). \quad (14)$$

The off-diagonal elements $Y_{21} = Y_{12}$ and $Y_{45} = Y_{54}$ correspond to the conductivity (p.u.l.) between the rails of the first and second tracks, respectively. Since conductivity between the overhead line and rails under normal operation conditions is negligible, the corresponding elements of the admittance matrix were assumed to be zero: $\underline{Y}_{5i} = \underline{Y}_{i5} = 0$, $\underline{Y}_{6i} = \underline{Y}_{i6} = 0$ ($i = 1 \dots 4$).

A multiconductor traction network is a distributed circuit with the voltage-current relations in a form

$$\frac{d\bar{\underline{U}}(x, f)}{dx} = -\underline{Z}(f)\bar{\underline{J}}(x, f), \quad (15)$$

$$\frac{d\bar{\underline{J}}(x, f)}{dx} = -\underline{Y}(f)\bar{\underline{U}}(x, f), \quad (16)$$

where $\bar{\underline{J}}(x, f) = \{\bar{I}_i(x, f)\}$ is the vector of harmonic currents of frequency f which components $\bar{I}_i(x, f)$, correspond to the currents in i -th line ($i = 1 \dots N$) and $\bar{\underline{U}}(x, f) = \{\bar{U}_i(x, f)\}$ is the vector of voltages with frequency f which components $\bar{U}_i(x, f)$, correspond to the voltages between lines i -th ($i = 1 \dots N$) and earth, x represents distance along the line.

The boundary conditions for the differential equations (15, 16) depend on the type of traction system, its arrangement and the trains operation modes.

The voltage and current of the harmonic of frequency f in rails at a point with coordinate

x are presented in usual for telegraph equations form as

$$\begin{bmatrix} \bar{\dot{U}}(x) \\ \bar{\dot{J}}(x) \end{bmatrix} = \begin{bmatrix} ch(\Gamma x) - sh(\Gamma x) \underline{Z} \\ \underline{Y}_c sh(\Gamma x) - \underline{Y}_c ch(\Gamma x) \underline{Z} \end{bmatrix} \begin{bmatrix} \bar{U}_1 \\ \bar{J}_1 \end{bmatrix},$$

where \bar{U}_1, \bar{J}_1 are the vectors of harmonics currents and voltages with frequency f at the point with $x=0$ (at the ESS terminals), $\underline{Z} = \underline{c} = \sqrt{\underline{Z}/\underline{Y}}$ is the matrix of the characteristic (wave) impedances of the lines, and $\underline{\Gamma} = \sqrt{\underline{Z}\underline{Y}}$ is the matrix of the propagation constants.

Traction current harmonics from single train in feeder zone

To evaluate influence of traction return current harmonics on track circuits it is necessary to determine the voltages and currents of harmonics in both rails of one track at the point where train receiver is connected and for the frequencies which is in TC's operation frequency range. Taking into account of longitudinally non-uniformity of traction network along feeding zone (transformers, TC's transmitters, receivers, cross-bonds, and etc. that are connected to the rails) it is expedient to represent two rails of one track as a single line with equivalent longitudinal parameters and then to determine the differential current of harmonics in two rails directly at required part of railway on the basis of electrical asymmetry of rails.

The disturbing vehicles are modeled as a sinusoidal current sources with several set of frequencies that represented by current vector $\mathbf{J}_H = \{\dot{J}_{Hi}(f_i)\}$ which components correspond to currents with frequencies f_i injected into rails at a point of train location.

Depended of simulation aim the values of harmonics $\dot{J}_{Hi}(f_i)$ at frequency f_i are taken as values measured during train tests or as the maximum interference values according to norms [1, 2].

A simplified three-wire equivalent circuit of the traction network with single train is shown in fig. 2.

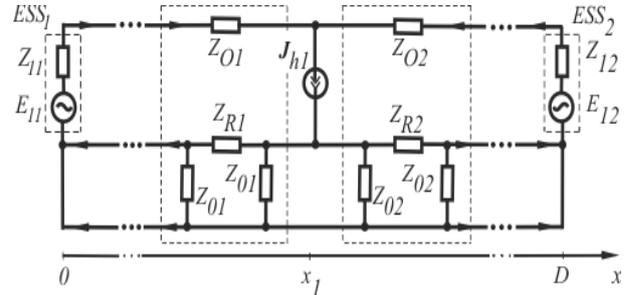


Fig. 2. A simplified three-wire equivalent circuit of the traction network for single train

Traction return current harmonics from train $\dot{J}_n(f_i)$ is divided in rails in two currents ($\dot{J}_n(f_i) = \dot{I}_{n1}(f_i) + \dot{I}_{n2}(f_i)$) which flow to ESS1 as $\dot{I}_1(f_i)$ and to ESS2 as $\dot{I}_2(f_i)$. These currents are inversely proportional to the impedances of two parts of the traction network $\underline{Z}_{TS1}(f_i)$ and $\underline{Z}_{TS2}(f_i)$ placed on the both side from the train

$$\frac{\dot{I}_{n1}(f_i)}{\dot{I}_{n2}(f_i)} = \left(\frac{\underline{Z}_{TS1}(f_i)}{\underline{Z}_{TS2}(f_i)} \right)^{-1}. \quad (17)$$

The distributions of the voltage and current of the traction return current harmonic in the rails between ESS and train for three-wire traction circuit have simple analytical form

$$\dot{U}_{Hi}(x, f) = \underline{C}_{Hi} \left[e^{\gamma(x-L-2l_2)} + e^{-\gamma(x-L_2)} \right], \quad (18)$$

$$\dot{I}_{Hi}(x, f) = -\frac{2\underline{C}_{Hi}}{\underline{Z}_{ci}} \left[e^{\gamma(x-L-2l_2)} - e^{-\gamma(x-L_2)} \right], \quad (19)$$

$$\underline{C}_{Hi} = \frac{\underline{C}_{1Hi}e^{\gamma L} + \underline{C}_{2Hi}e^{-\gamma L}}{1 + e^{-2\gamma l_2}}, \quad (20)$$

where $C1$ and $C2$ are constants of integration that are defined from boundary conditions, \underline{Z}_{ci} is the characteristic (wave) impedance of the lines, γ is the propagation constants, L is the distance between train and substation, l_2 is the spreading distance of the harmonic of the reverse traction current beyond the SSE.

Traction current harmonics from multiple trains in feeder zone

The equivalent circuit of a traction system with two ESS and multiple trains is shown in fig. 3.

The distribution of traction return current harmonics in rails generated by multiple trains in feeder zone are determined by using superposition method. On first stage of this method the partial currents in all branches of the circuit caused by one electromotive force (EMF) or current source are calculated with assumption that the other EMF sources are replaced by short-circuited branches and current sources are replaced by breaks in the circuit. Then the total currents in each of the branches are determined as a sum of the partial currents.

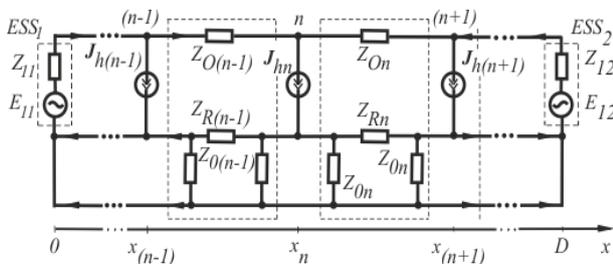


Fig. 3. The equivalent circuit of a traction system with multiple trains (only two trains with numbers n and $(n+1)$ are shown)

On the basis of the established mathematical model, a computer program have been developed

Results

To illustrate the application of the developed computer model, the distribution of the traction return current harmonics was computed for AC direct feeding traction network 2x25 kV electric railway system with two-side ESSs. The parameters of the model have been set as follows: distance between substations D_1 is 40 km, number of trains N_{T1} is varied from 1 to 5, the trains' coordinates are $x_n = 6, 13, 20, 27, 34$ km, the electrical conductivity of the earth σ_g is varied from 10^{-2} to 10^{-3} Sm/km, the rail-to-earth conductivity σ_{re}

is varied from 0.02 to 1 Sm/km, the harmonic's current at 25 Hz, generated in rails by each of the trains is 1 A (RMS) (that corresponds to maximum value of interference in the rails at 25 Hz [4]).

The maximum interference from trains is in the areas nearest to trains and also to the point of connection of return feeder to rails (at ESS terminals). Therefore, simulation results are represented for the traction return network area between ESS1 ($x=0$) and the train ($x=6$ km) (fig. 4). The traction harmonic current in rails are increased with increasing of train number in feeder zone and with decreasing of the rail-to-earth conductivity.

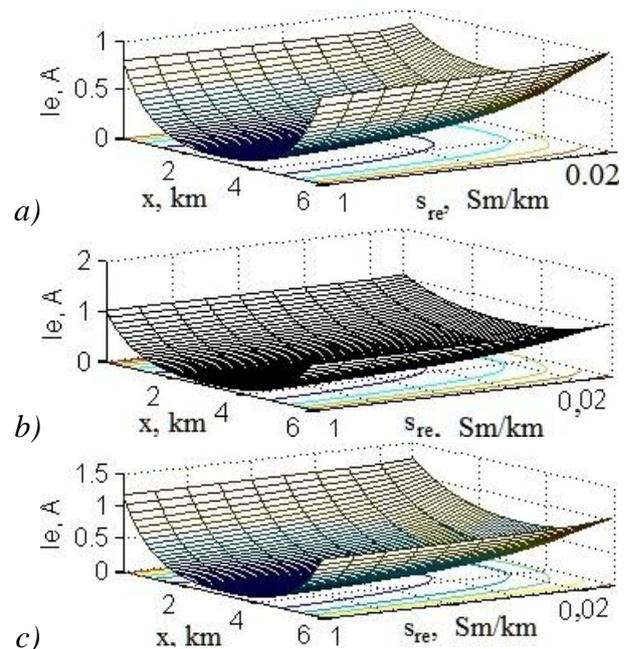


Fig. 4. Dependence of the harmonic current at a frequency 25 Hz in the rails from the coordinate x and the rail to earth conductivity σ_{re} for one (a), two (b) and five (c) locomotives in feeder zone

In case of one locomotive in feeder zone the interference at 25 Hz in the rails near the ESS don't exceed a limit value of 1 A even in unfavorable operation conditions for the rail-to-earth conductivity equal 0.02 Sm/km. (fig. 4 a). If number of trains in feeder zone increased (from 1 to 5) the interference at 25 Hz also increased and it values at rail-to-earth conductivity equal 0.02 Sm/km reach to 1.073 A for two locomotives (at $x=6$ and

13 km) and 1.233 A for five locomotives (fig. 4 (b), (c)). These values of interference are exceed the limit value 1 A at 25 Hz. Similar results are obtained for traction harmonic currents at frequencies 420, 480, 580, 720, 780 Hz.

Conclusion

A mathematical and computer model of traction current harmonic distribution in direct feeding traction network with multiple vehicles in feeder zone has been established. The traction network was represented as series-connected multipoles, corresponded to longitudinally uniform network areas which are modeled with MTL equations.

The model has been simplified as follows. The lines with equal or close to each other potentials are represented as a single line with equivalent electrical parameters. The disturbing vehicles are modeled as sinusoidal current sources with several set of frequencies that are represented by current vector $\mathbf{J}_H = \{J_{Hi}(f_i)\}$ which components correspond to currents with frequencies f_i injected into rails. Only return current harmonics with frequencies that lie in frequency range of track circuit receiver were considered. Depended of simulation aim the values of harmonics $J_{Hi}(f_i)$ at frequency f_i are taken as values measured during train tests or as the maximum interference values according to norms.

The distribution of the traction return current harmonics was computed for direct feeding traction network 1x25 kV AC electric railway system with two-side ESS and with 1 to 5 vehicles in feeder zone.

The maximum interference from trains is in the areas nearest to trains and also to the point of connection of return feeder to rails (at ESS terminals). The traction harmonic current in rails are increased with increasing of train number in feeder zone and with decreasing of the rail-to-earth conductivity.

The interference at 25 Hz in the rails area near the ESS for one locomotive in feeder

zone don't exceed a limit value of 1 A even in unfavorable operation conditions for the rail-to-earth conductivity equal 0.02 Sm/km. If number of trains are increased (from 1 to 5) the interference at 25 Hz also increased and it values at rail-to-earth conductivity equal 0.02 Sm/km reach to 1.073 A for two locomotives and to 1.233 A for five locomotives, that exceed the limit value of the interference in rails at 25 Hz.

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- Ключові слова:** електромагнітна сумісність, математична та комп'ютерна модель розподілення гармонік тягового струму, тягова мережа, фідерна зона, рухомий склад нових типів.
- Ключевые слова:** электромагнитная совместимость, математическая и компьютерная модель распределения гармоник тягового тока, тяговая сеть, фидерная зона, подвижной состав новых типов.
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