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THE COMPARATIVE ANALYSIS OF CALCULATING METHODS FOR AC IMPEDANCE OF R65 TYPE RAILS AND TRACK 1520 MM GAUGE IN THE AUDIO FREQUENCY RANGE

Introduction

Knowledge of the exact AC rail impedance in a wide frequency range is necessary for designing and modeling audiofrequency track circuits [1, 2], determining of the return traction current distribution in rails of high-speed railways [3, 4], testing new types of vehicles on electromagnetic compatibility with track circuits [5, 6], assuring of induction coupling between jointless track circuits and track-circuit-reader antenna [7], etc. Results of the rail impedance investigations were published in papers of Carpenter and Hill [8, 9], Mariscotti and Pozzobon [10, 11]. A theoretical analysis of frequency dependence of rail impedance is based mainly on the so-called Carson/Pollaczek model or simply Carson's method that was proposed almost simultaneously by Carson [12] and Pollaczek [13] for determining the AC transmission line impedance considering earth return current. They obtained equations for impedance under such specific assumptions:

– conductors are parallel and infinite length;

– ground under wires is homogeneous and of constant resistivity;

– dielectric and magnetic permittivity of the ground are considered to be equal to unity;

– the displacement currents in the air and the ground are neglected.

Solutions of the equations for AC rail impedance in [12, 13] were obtained for quasi static transverse electromagnetic modes (TEM) of an electromagnetic field (i.e. in assumption that electromagnetic field include only longitudinal modes).

Based on Carson's work, Wise [14, 15] proposed more general expression considered the displacement currents in the air and the ground. Sunde [16] summarized these works and proposed generalized formula. Later some approximate expressions were proposed by Gary [17], Deri et al [18], Rachidi [19] and others.

It has been shown in [19] that the validity of the Carson's approximation extends to frequencies of about a few MHz for typical overhead power lines and for earth conductivity of about 0.01 Sm/m. Theoretical consideration of the frequency dependence of running rails impedance is more complicated task due to:

– complex structure of the railway track consisting of two rails, sleepers, ballast;

– skin-effect in rails;

– strong current dependence of magnetic permeability of rail steel;

– influence of the nearly lied ground due to displacement current being inducted in the ground and leakage current due to small resistance between rail and a ground;

– complex shape of rail cross-cut.

Available data of rail impedance investigations [8–-11] are related mainly to the UIC 60 rail type with 1435 mm track. Data for electric impedance of rails of R65 type and 1520 mm track are given in table 1 [20].

The frequency dependences of real and imaginary part of rail impedance plotted according to [20] look like broken line, probably due to a measurement error.

If to plot frequency dependence of rail impedance calculated according to Carson's method at the same axes as data [20], one can see differences between these plots especially for frequencies greater than 1 kHz.

Despite the practical need to have a frequency dependence of the impedance of P65 type rails and track 1520 mm gauge in audiofrequency range ($10^2 \ldots 10^4$ Hz), few studies on this issue are known.

The aim of the work is to perform a comparative analysis of the methods for calculating of the AC impedance of the R65 type rails of a track 1520 mm gauge in the audiofrequency range.

Table 1

Traction rail impedance [20]

f, Hz	*Z*, Ohm/km	*f*, Hz	*Z*, Ohm/km
25	0.308+0.394i	580	1.077+6.106i
50	0.338+0.725i	720	1.221+7.299i
75	0.401+0.992i	780	1.236+7.803i
175	0.618+1.902i	4545	1.529+43.773i
420	0.935+4.810i	5000	1.700+48.670i
480	0.938+5.318i	5555	1.871+53.567i

Mathematical formulation

A traction system of electrified railways is a system of wires (lines) with spatially distributed parameters which voltage-current relations mathematically described on the base of the multiconductor transmission line theory as matrix differential equations

$$\frac{\partial \dot{V}(\omega, x)}{\partial x} = -\underline{Z}(\omega)\dot{I}(\omega); \qquad (1)$$

$$\frac{\partial \dot{I}(\omega, x)}{\partial x} = -\underline{Y}(\omega)\dot{V}(\omega); \qquad (2)$$

where $\underline{Z}$ is a matrix of series impedances per unit length (p.u.l.), and $\underline{Y}$ is a matrix of shunt admittances (p.u.l.), x is a coordinate along the line, $\omega = 2\pi f$ is a cyclic frequency. The matrices $\underline{Z}$ and $\underline{Y}$ are always symmetric.

Series Impedance

At low and medium frequencies ($\le 10^4$ Hz) the rail series impedance is given by

$$\underline{Z}(\omega) = \underline{Z}_C(\omega) + \underline{Z}_E(\omega) + \underline{Z}_G(\omega), \qquad (3)$$

where $\underline{Z}_C(\omega)$ is the internal rail impedance that depends on frequency due to skin-effect; $\underline{Z}_E(\omega)$ is the external impedance and $\underline{Z}_G(\omega)$ is the ground-return impedance of the conductor.

For one track railway that consists of two rails above lossy ground the diagonal elements of a series impedance matrix $\underline{Z}_{ii}$ $(i = 1, 2)$ are the values of a rail self-impedance (p.u.l.) defined as the ratio of the voltage drop (p.u.l.) to the current flowed in the rail and returned through the earth.

Off-diagonal elements of a series impedance matrix $\underline{Z}_{ij}$ $(i, j = 1, 2;\ i \ne j)$ are mutual impedances between i-th and j-th conductors defined as the ratio of the induced voltage (p.u.l.) in i-th conductor to the current in j-th conductor. Both the self and mutual impedances are affected by the earth return current.

Self impedance

Self impedance of the conductor is a sum of two components

$$\underline{Z}_{ii} = \underline{Z}_{Cii} + \underline{Z}_{Eii}, \qquad (4)$$

where $\underline{Z}_{Cii}$ is the internal impedance of the conductor and $\underline{Z}_{Eii}$ is the external impedance of the conductor that is given by

(2)

$$\underline{Z}_{Eii} = \underline{Z}_{Egii} + \underline{Z}_{gii}, \qquad (5)$$

where $\underline{Z}_{Egii}$ is the geometric term due to reactance involved in the magnetic field in the air and $\underline{Z}_{gii}$ is the ground-return impedance of the conductor.

The external self-impedance of the conductor is equal $\underline{Z}_{Egii} = j\omega L_{ii}$, where L_{ii} is an external inductance of the conductor that is given by

$$L_{ii} = \frac{\mu_0}{2\pi} \ln \frac{2h_i}{r_i}, \qquad (6)$$

where $\mu_0 = 4\pi 10^{-7}$ H/m is the magnetic permeability constant, h_i is the height of the conductor above the ground and r_i is the radius of the conductor.

The ground-return impedance of the conductor can be written as

$$\underline{Z}_{gii} = R_{gii} + X_{gii}. \qquad (7)$$

The mutual impedance

The mutual impedance $\underline{Z}_{ij}$ of two conductors i and j can be written as

$$\underline{Z}_{ij} = \underline{Z}_{mij} + \underline{Z}_{gij}, \qquad (8)$$

where $\underline{Z}_{mij} = j\omega L_{ij}$ is the impedance due to mutual inductance L_{ij} between two conductors and $\underline{Z}_{gij}$ is the impedance of the earth return path. The mutual inductance L_{ij} is defined by geometric parameters of the line conductor system (fig. 1) and can be written as

$$L_{ij} = \frac{\mu_0}{2\pi}\ln\frac{D_{ij}'}{D_{ij}} = \frac{\mu_0}{2\pi}\ln\sqrt{\frac{d_{ij}^2 + \left(h_i + h_j\right)^2}{d_{ij}^2 + \left(h_i - h_j\right)^2}}, \qquad (9)$$

where D_{ij} is the distance between conductors i and j, and D_{ij}' is the distance between conductor i and the image of conductor j. The meanings of the other geometric parameters from (9) are illustrated in fig. 1.

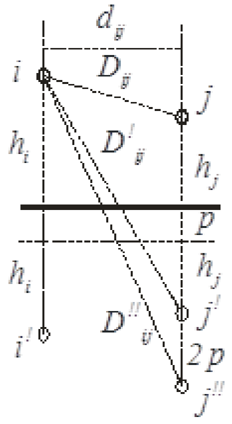

Fig. 1. Geometry of conductors and their images

The ground-return impedance (due to the earth influence) can be written as

$$Z_{Gij} = R_{Gij} + jX_{Gij}. \qquad (10)$$

The internal impedance of a conductor in (4) is given by

$$Z_{Cii} = R_{Cii} + jX_{Cii}, \qquad (11)$$

where R_{Cii} is the internal resistance and $X_{Cij} = \omega L_{Cii}$ is the internal reactance.

Under the assumption that current density is uniformly distributed at cross-section of conductor the internal self-resistance and inductance are given by well-known simple formulae

$$R_{Cii} = \left(\sigma S\right)^{-1}, \qquad (12)$$

$$L_{Cii} = \frac{\mu\mu_0 r_i}{8\pi}, \qquad (13)$$

where σ is the conductivity of a conductor, S is the area of the conductor cross-section, μ is relative magnetic permeability.

The radius of an equivalent circular cross-section (geometric mean radius) of a rail may be evaluated with two expressions [11, 21]. For low frequencies, when the current is distributed almost uniformly across the rail cross-section, equivalent radius may be computed as

$$r_i = \sqrt{S/\pi}. \qquad (14)$$

For high frequencies, when the current is distributed almost along the conductor perimeter, equivalent radius may be computed as

$$r_i = P_r/2\pi, \qquad (15)$$

where P_r is a perimeter of rail cross-section.

Expressions (12)–(14) doesn't provide accurate estimation of rail internal self-resistance and inductance due to a complex shape of rail cross-section, the skin effect at AC and varying magnetic permeability with current value variation. For more precision estimation of a rail internal self-impedance a finite-element method (FEM) have been proposed [8].

In present work for calculation of rail internal impedance an approximation method with representation of the rail as equivalent cylindrical conductor with taking into account of skin effect has been used [21].

According to this method the internal impedance of a cylindrical conductor is given by

$$\underline{Z}_{Cii} = \frac{\rho m}{2\pi r} \frac{\alpha J_0(\alpha m r)}{J_1(\alpha m r)} =$$

$$= \frac{\rho m}{2\pi r} \frac{\mathrm{ber}\left(\sqrt{2}\, r/\delta\right) + j\mathrm{bei}\left(\sqrt{2}\, r/\delta\right)}{\mathrm{ber}'\left(\sqrt{2}\, r/\delta\right) + j\mathrm{bei}'\left(\sqrt{2}\, r/\delta\right)}; \quad (16)$$

$$m = \sqrt{\frac{\omega\mu\mu_0}{\rho}}; \quad (17)$$

$$\alpha = \exp(j3\pi/4), \quad (18)$$

$$\delta = \left(\sqrt{\pi f \mu\sigma}\right)^{-1} \quad (19)$$

where δ is a depth of penetration (of scin-effect), ρ is a resistivity of the conductor, J_ν is the Bessel function of first kind and order ν, ber, bei, ker and kei are Kelvin's functions which belong to the Bessel function family, and ber′, bei′, ker′ and kei′ are their derivatives. Kelvin's functions are often defined as

$$\mathrm{ber}(x) = 1 - \frac{x^4}{2^4 (2!)^2} + \frac{x^8}{2^8 (4!)^2} - \ldots; \quad (19)$$

$$\mathrm{bei}(x) = \frac{x^2}{2^2} - \frac{x^6}{2^6 (3!)^2} + \frac{x^{10}}{2^{10} (5!)^2} - \ldots; (20)$$

and

$$\mathrm{ber}(q) + j\mathrm{bei}(q) = J_0\left(j^{-1/2}\right). \quad (21)$$

Active and reactive resistances of a cylindrical conductor are given by

$$R_{Cii} = \frac{R_S}{\sqrt{2\pi r_i}} \times$$

$$\times \frac{\mathrm{ber}(q)\mathrm{bei}'(q) - \mathrm{bei}(q)\mathrm{ber}'(q)}{\left(\mathrm{ber}'(q)\right)^2 + \left(\mathrm{bei}'(q)\right)^2}; \quad (22)$$

$$X_{Cii} = \frac{R_S}{\sqrt{2\pi r_i}} \times$$

$$\times \frac{\mathrm{ber}(q)\mathrm{ber}'(q) - \mathrm{bei}(q)\mathrm{bei}'(q)}{\left(\mathrm{ber}'(q)\right)^2 + \left(\mathrm{bei}'(q)\right)^2}, \quad (23)$$

$$q = \sqrt{2r_i/\delta}, \quad (24)$$

where $R_S = (\sigma\delta)^{-1}$ is a surface resistance.

Earth Contributions

The correction terms for the self-impedance of i-th conductor and mutual impedance of two conductors i and j due to earth path impedance are obtained by Carson method [12]:

$$\underline{Z}_{Gii} = \frac{j\omega\mu}{\pi} \int_0^\infty \frac{\exp(-2h_i\xi)}{\xi + \sqrt{\xi^2 + j\omega\mu\sigma}} d\xi, \quad (25)$$

$$Z_{Gij} = \frac{j\omega\mu}{\pi} \int_0^\infty \frac{\exp\left[-\left(h_i + h_j\right)\xi\right]}{\xi + \sqrt{\xi^2 + j\omega\mu\sigma}} d\xi. \quad (26)$$

More general expression for mutual ground impedance between two conductors i and j applicable to higher frequency band derived by Sunde [16] is given by

$$Z_{Gij} = \frac{j\omega\mu}{\pi} \times$$

$$\times \int_0^\infty \frac{\exp\left[-\left(h_i + h_j\right)\xi\right]}{\xi + \sqrt{\xi^2 + \gamma_g^2}} \cos\left(d_{ij}\xi\right) d\xi, \quad (27)$$

where γ_g is a wave propagation constant defined as

$$\gamma_g = \sqrt{j\omega\mu_0\left(\sigma_g + j\omega\varepsilon_0\varepsilon_g\right)}. \quad (28)$$

These expressions contain infinite integrals with complex arguments. For theirs evaluation Carson has proposed infinite series

$$R_{Gii} = 4\omega 10^{-7} \left\{\frac{\pi}{8} - b_1 k + b_2 \left(C_2 - \ln k\right) k^2 + \right.$$

$$\left. + b_3 k^3 - d_4 k^4 - \ldots \right\}; \quad (29)$$

$$X_{Gii} = 4\omega 10^{-7}\left\{\frac{1}{2}\left(0.6159315-\ln k\right)+b_1 k + \right.$$
$$\left. + d_2 k^2 + b_3 k^3 - b_4\left(C_4-\ln k\right)k^4+\ldots\right\}; \quad (30)$$
$$R_{Gij} = 4\omega 10^{-7}\left\{\frac{\pi}{8} - b_1 k_m \cos\theta + b_2\left[\left(C_2 - \right.\right.\right.$$
$$\left.\left. -\ln k_m\right)k_m{}^2\cos 2\theta + \theta k_m^2 \sin 2\theta\right] +$$
$$\left. + b_3 k_m{}^3 \cos 3\theta - d_4 k_m{}^4 \cos 4\theta - \ldots\right\}; \quad (31)$$
$$X_{Gij} = 4\omega 10^{-7}\left\{\frac{1}{2}\left(0.6159315-\ln k_m\right)+b_1 k_m \times \right.$$
$$\times\cos\theta - d^2 k_m{}^2\cos 2\theta + b^3 k_m{}^3 \cos 3\theta - b_4\left[\left(C_4 - \right.\right.$$
$$\left.\left. -\ln k_m\right)k_m^4\cos 4\theta + \theta k_m^4 \sin 4\theta\right]\ldots\Big\}; \quad (32)$$

where

$$b_1 = \sqrt{2}/6;$$
$$b_2 = 1/16;$$
$$b_i = b_{i-2}\frac{sign}{i\left(i+2\right)};$$
$$C_1 = 1.3659315;$$
$$C_i = C_{i-2} + \frac{1}{i} + \frac{1}{i+2};$$
$$d_i = \frac{\pi}{4} b_i$$
$$\theta = \arcsin\left(\frac{d_{ij}}{D_{ij}}\right);$$
$$k = 4\pi\sqrt{5}\ 10^{-4} 2h_i\sqrt{f\sigma_g};$$
$$k_m = 4\pi\sqrt{5}\ 10^{-4} D_{ij}'\sqrt{f\sigma_g}.$$

These approximations are valid for a limited range of frequencies, and medium frequencies are not covered [21].

The complex image method

The complex image method (complex depth of earth return method) [17, 18] assumes that the current in conductor i returns through an imagined earth path located directly under the original conductor at a depth of $\left(h_i + 2p\right)$ as shown in fig. 1, where p is the skin depth of the ground. Thus, the self and the mutual impedances can be written as

$$\underline{Z}_{Eii} = j\omega\frac{\mu_0}{2\pi}\ln\frac{2\left(h_i+p\right)}{r_i}; \quad (33)$$
$$Z_{ij} = j\omega\frac{\mu_0}{2\pi}\ln\frac{D_{ij}'}{D_{ij}} =$$
$$= j\omega\frac{\mu_0}{2\pi}\ln\sqrt{\frac{d_{ij}^2+\left(h_i+h_j+2p\right)^2}{d_{ij}^2+\left(h_i-h_j\right)^2}}; \quad (34)$$

where $p = \sqrt{\rho/j\omega\mu_0}$.

Results

The impedance of running rails of R65 type and track 1520 mm gauge have been calculated using formulas (3)-(35). The correction terms for the self- and mutual impedance of the rails due to earth path were determined by using two above considered methods - Carson's method with representation of integrals as infinite series (16)–(25) and the complex image method (34), (35). Calculations have been performed for next parameters of tracks: perimeter of rail cross-section is P_r =0,7 m, distance between axes of one track rails is $d_{ij} = 1.6$ m, height rails above a ground is $h_i = 0.5$ m, conductivity of the ground is $\sigma_g = 0.1$ Sm/m, steel resistivity is $\sigma_S = 0.21$ Ohm mm^2/m, steel relative permeability is $\mu = 100$.

The rail loop electrical parameters (p.u.l.) are obtained as

$$R = \left(R_1 + R_2\right) = 2R_1; \quad (35)$$
$$L = 2\left(L_1 - L_{12}\right); \quad (36)$$

where $R_1 = R_2$ and $L_1 = L_2$ are, respectively, self-resistance and self-inductance of rails, L_{12} is mutual inductance of rails.

Frequency dependences of the calculated active resistance R and inductance L of rail loop (p.u.l.) with rails of R65 type and track of 1520 mm gauge are shown in fig. 2.

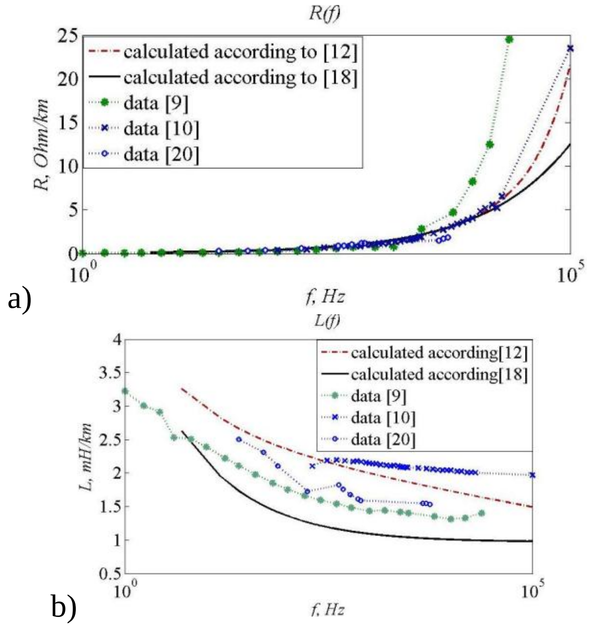

Fig. 2. Frequency dependences of the resistance R *(a)* and inductance L *(b)* (p.u.l.) of rail loop for R65 type rails and track of 1520 mm gauge

In fig. 2 the frequency dependences of the rail loop resistance and inductance (p.u.l.) of R65 type rails and track of 1520 mm gauge according to [20] are also plotted (as zigzag-form lines).

Values of rail loop resistance R calculated by Carson's method [12] and the complex image method [18] are in good agreement with the data [20], while the values of inductance L calculated by these two methods ([12] and [18]) are differ in values from each other and from data [20].

The resistance and inductance of the running rails of track 1435 mm gauge [8-11] are also represented in fig. 2 for comparison. Data [8] were measured for railway section of 36 m length with concrete and wooden sleepers in 1 Hz…25 kHz frequency range.

Serial rail loop resistance and inductance were measured in [10] for loop circuit with rails of UNI 60 type, and area of cross-section – 7679 mm^2 with full length – 5.8 m and length between voltage terminals – 5.2 m. The data of [10] were recalculated on 1 km rail length and also shown in fig. 2.

Since data [8–11] have been obtained for the other types of track, rails, sleepers, etc., these data were presented for qualitative comparison with data for R65 type rails and track of 1520 mm gauge (fig. 2).

Therefore, it may be concluded that the AC running rail impedance obtained by Carson's method [12] and the complex image method [18] are in good qualitative agreement with the data of [8–11] for rails of UIC 60 type in frequency range $10^0 \ldots 10^5$ Hz.

The results of calculation for rails of R65 type and track of 1520 mm gauge according to Carson's method and complex image method differ from data in [20], and these differences increase with increasing of frequency. Such behavior may be due to error of calculation methods caused by small height of rails above lossy ground ($h_i \le 1$ m) and high electrical conductivity between rails and ground.

Observed miscalculation of the AC impedance of R65 type rails for frequencies $> 10^3$ Hz can be eliminated by using correction factors in calculation expressions.

Conclusion

In this work the comparative analysis of the calculating methods for AC impedance of R65 type rails of a track 1520 mm gauge in the audiofrequency range have been performed.

Results of AC rail impedance measurements and theoretical description of frequency dependence of impedance for wires above lossy ground are briefly reviewed. Mathematical formulation of Carson's method and the complex image method proposed by Deri with coworkers for impedance of transmission lines were represented. The impedance of running rails of R65 type and track of 1520 mm gauge have been calculated by using Carson's method and the complex image method. Due to the insufficient amount of literature data concerning AC impedance measurements for rails of type R65 of 1520 mm gauge, the results of calculations for these rails were compared with the measured values for both rails of type R65, 1520 mm gauge, as well as for rails of type UIC 60 with 1435 mm gauge. Calculated frequency dependencies of the AC impedance of R65 type rails are in good qualitative agreement with literature data for rails UIC 60 in frequency range $10^0 \ldots 10^5$ Hz.

The results for rails of R65 type and 1520 mm gauge calculated according to Carson's method and complex image method differ from reference data presented in literature, and these differences increase with increasing of frequency. Such behavior may be due to error of calculation methods caused by small height of rails above lossy ground and high electrical conductivity between rails and a ground.

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Ключові слова: імпеданс рейки , тональні рейкові кола, метод Карсона, метод комплексного зображення, тональна частота.

Ключевые слова: импеданс рельса, тональные рельсовые цепи, метод Карсона, метод комплексного изображения, тональная частота.

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